

C.U.SHAH UNIVERSITY

Winter Examination-2015

Subject Name : Advance Mathematics

Subject Code : 2TE02AMT2

Branch : Diploma(All)

Semester : 02

Date : 19/11/2015

Time : 10:30 am To 1:30 pm

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1

Attempt the following questions:

(14)

- a) Magnitude of $3i - 4j - 5k$ is _____.
 (a) $2\sqrt{5}$ (b) 0 (c) -6 (d) $5\sqrt{2}$
- b) If $\mathbf{a} = \mathbf{i} + \mathbf{j}$ and $\mathbf{b} = \mathbf{j} - \mathbf{k}$ then $\mathbf{a} \cdot \mathbf{b} =$ _____.
 (a) 0 (b) -1 (c) 1 (d) none of these
- c) If $\mathbf{a} = \mathbf{i} - \mathbf{j} + \mathbf{k}$ then $\hat{\mathbf{a}} =$ _____.
 (a) $\frac{1}{\sqrt{2}}(1, -1, 1)$ (b) $\frac{1}{\sqrt{3}}(1, -1, 1)$ (c) $\sqrt{3}$ (d) none of these
- d) $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{4n} =$ _____.
 (a) e^4 (b) e (c) 1 (d) none of these
- e) $\lim_{x \rightarrow 0} \frac{\sin \sqrt{x}}{\sqrt{x}} =$ _____.
 (a) 0 (b) e (c) $\frac{1}{2}$ (d) 1
- f) $\lim_{x \rightarrow 0} \frac{5^x - 1}{x} =$ _____.
 (a) 0 (b) e^5 (c) $\log_e 5$ (d) 1
- g) If $f(x) = x^2 - 1$ then $f(-1) =$ _____.
 (a) -2 (b) 0 (c) -1 (d) none of these
- h) $\frac{d(6^x)}{dx} =$ _____.
 (a) $6^x \log_e 6$ (b) $\log_e 6$ (c) $x^6 \log_e 6$ (d) $6^x \log_e x$



i) $\frac{d(\sqrt{x})}{dx} = \underline{\hspace{2cm}}$
 (a) $2x^{\frac{1}{2}}$ (b) $\frac{1}{2}\sqrt{x}$ (c) $\frac{2}{\sqrt{x}}$ (d) $\frac{1}{2\sqrt{x}}$

j) $\frac{d(x^e)}{dx} = \underline{\hspace{2cm}}$
 (a) x^{e-1} (b) xe^{x-1} (c) ex^{e-1} (d) e^{x-1}

k) $\int e^x dx = \underline{\hspace{2cm}}$
 (a) $\log x$ (b) e^x (c) $\frac{e^x}{\log x}$ (d) none of these

l) $\int \frac{1}{x^2+1} dx = \underline{\hspace{2cm}}$
 (a) $\tan^{-1} x + c$ (b) $\cos^{-1} x + c$ (c) $\cot^{-1} x + c$ (d) $\sin^{-1} x + c$

m) $\int \frac{1}{\sqrt{1-x^2}} dx = \underline{\hspace{2cm}}$
 (a) $\tan^{-1} x + c$ (b) $\cos^{-1} x + c$ (c) $\cot^{-1} x + c$ (d) $\sin^{-1} x + c$

n) $\int_0^1 x^2 dx = \underline{\hspace{2cm}}$
 (a) 2 (b) 1 (c) $\frac{1}{3}$ (d) none of these

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions (14)

a) If $a = i + 2j - k$, $b = 3i + j + 2k$ and $c = -2i - j + 5k$ then find $|2a + 3b - c|$. (5)

b) If $a = 2i - j$, $b = i + 3j - 2k$ then obtain $|(a + b) \times (a - b)|$. (5)

c) For what value of p, vectors $2i + 3j - k$ and $pi - j + 3k$ are perpendicular to each other? (4)

Q-3 Attempt all questions (14)

a) A particle moves from the point $3i - 2j + k$ to the point $i + 3j - 4k$ under the effect of constant forces $i - j + k$, $i + j - 3k$ and $4i + 5j - 6k$. Find the work done. (5)

b) Prove that angle between two vectors $i + 2j$ and $i + j + 3k$ is $\sin^{-1}\left(\sqrt{\frac{46}{55}}\right)$. (5)

c) Find unit vector which is perpendicular to $a = 5i + 7j - 2k$ and $b = 3i + j - 2k$. (4)

Q-4 Attempt all questions (14)

a) Prove that $\lim_{x \rightarrow 3} \frac{\sqrt{x+2} - \sqrt{5}}{\sqrt{x+4} - \sqrt{7}} = \frac{\sqrt{35}}{5}$ (5)

b) Evaluate: $\lim_{\theta \rightarrow 0} \frac{\operatorname{cosec}\theta - \cot\theta}{\theta}$ (5)



- c) Prove that If $f(x) = \log\left(\frac{x-1}{x}\right)$ then prove that $f(x) + f(-x) = f(x^2)$. (4)

Q-5 Attempt all questions (14)

- a) Find derivative of $f(x) = x^3$ using definition. (5)
- b) Find $\frac{dy}{dx}$ if $y = \log\left[x + \sqrt{x^2 + a^2}\right]$. (5)
- c) Find $\frac{dy}{dx}$ if $y = \frac{a + b\sin x}{a\sin x + b}$. (4)

Q-6 Attempt all questions (14)

- a) Find $\frac{dy}{dx}$ if $e^x + e^y = e^{x+y}$. (5)
- b) The equation of motion of a particle is $S = 2t^3 + 3t^2 - 12t + 5$. Find velocity at $t = 0$ and acceleration at $t = 1$. (5)
- c) Find $\frac{dy}{dx}$ if $y = (\sin x)^{\tan x}$. (4)

Q-7 Attempt all questions (14)

- a) Evaluate: $\int \frac{(1-3x)^2}{x^3} dx$ (5)
- b) Evaluate: $\int x \log x dx$ (5)
- c) Evaluate: $\int \sin^4 x \cos x dx$ (4)

Q-8 Attempt all questions (14)

- a) Prove that $\int_0^{\frac{\pi}{2}} \frac{\sec x}{\sec x + \operatorname{cosec} x} dx = \frac{\pi}{4}$ (7)
- b) Find the area of standard ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (7)

